Moral Hazard and Insurance

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1 Introduction

In this teaching note, we analytically explore the incentives for risk management in the absence and presence of insurance, and the role of contract design in ensuring that consumers have proper incentives to mitigate risk even when they are insured. We assume that there are only two possible states of the world - loss and no loss. We'll also assume (for the sake of simplicity and without loss of generality) that the consumer is risk neutral and therefore interested in maximizing the expected value of wealth.[1]

Our consumer needs to determine whether to invest in safety. The cost of investing in safety is $c(s)$, where s corresponds to the level of safety chosen and $c'(s) > 0$; i.e., higher safety leads to higher cost. The probability of loss $p(s)$ is inversely related to the level of safety chosen; i.e., $p'(s) < 0$.

2 Optimal safety in the absence of insurance

In the *absence* of insurance, expected wealth ($E(W)$) is written as

$$E(W) = W_0 - c(s) - p(s)L. \tag{1}$$

Next, we maximize $E(W)$ by differentiating equation (1) with respect to s and solving for

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[1]While the assumption of risk neutrality begs the question as to why there would be a demand for insurance in the first place, this assumption helps to more clearly illustrate the role that contract design plays in incentivizing consumers to mitigate risk (in this case, by linking insurance premiums to investments in safety).

the optimal value of s (notated as s^*) that causes the resulting equation to be equal to zero:

$$\frac{dE(W)}{ds} = -c'(s^*) - p'(s^*)L = 0. \tag{2}$$

Rearranging equation (2), we obtain a very familiar result; the optimal level of safety s^* occurs when the marginal cost of safety (c'(s^*)) is equal to the marginal benefit of safety (-p'(s^*)L); i.e.,

$$\underbrace{c'(s^*)}_{\text{Marginal Cost}} = \underbrace{-p'(s^*)L}_{\text{Marginal Benefit}} \tag{3}$$

The optimal level of investment in safety when there is no insurance is shown in Figure 1. In this figure, the word "hedge" corresponds to the coinsurance rate. Thus, when there is no hedge (i.e., $\alpha = 0$), the optimal value for s^* is at point A, which is where the marginal benefit and marginal cost schedules intersect.

3 Optimal safety in the presence of (risk-insensitive) insurance

Next, we introduce insurance in which the insurer covers the proportion α of the risk for a premium of $\$\alpha P$. Initially, we assume that the insurance premium P is risk-insensitive in the sense that the premium is not adjusted according to risk management decisions made by the consumer. Thus, expected wealth is

$$E(W) = W_0 - c(s) - (1-\alpha)p(s)L - \alpha P, \tag{4}$$

and s^* is determined by the following equations:

$$\frac{dE(W)}{ds} = -c'(s^*) - (1-\alpha)p'(s^*)L = 0; \text{thus,} \tag{5}$$

$$\underbrace{c'(s^*)}_{\text{Marginal Cost}} = \underbrace{-(1-\alpha)p'(s^*)L.}_{\text{Marginal Benefit}} \tag{6}$$

Since coinsurance proportionately scales down the marginal benefit of safety, s^* is lower when insurance is purchased. Clearly, whenever $\alpha > 0$, then the presence of (risk-insensitive) insurance discourages investment in safety. For example, in Figure 1, if the coinsurance rate $\alpha = 0.5$, then the optimal s^* occurs at point B. Even worse, if the consumer purchases full coverage (i.e., $\alpha = 1$), then the marginal benefit associated with investing in safety goes to zero, which in turn results in an optimal value for s^* of zero. This is illustrated in Figure 1 where the marginal benefit schedule for the safety investment schedule lies on the X axis.

4 Optimal safety in the presence of (risk-sensitive) insurance

The solution to this dilemma is for the insurer to charge a risk-sensitive premium in the sense that the premium is a function of the *level of investment in safety*. In other words, let $P = P(s)$, where $P'(s)<0$. Thus equation (4) is rewritten as:

$$E(W) = W_0 - c(s) - (1 - \alpha)p(s)L - \alpha P(s) \tag{7}$$

and s^* is determined by the following equation:

$$\frac{dE(W)}{ds} = -c'(s^*) - (1 - \alpha)p'(s^*)L - P'(s^*) = 0 \tag{8}$$

Thus, the equilibrium condition of marginal cost equaling marginal benefit is written as follows:

$$\underbrace{c'(s^*)}_{\text{Marginal Cost}} = \underbrace{-(1 - \alpha)p'(s^*)L - \alpha P'(s)}_{\text{Marginal Benefit}} \tag{9}$$

In equation (9), even when α =1, it will be optimal to invest in safety, since the premium charged is sensitive to the level of investment in safety.

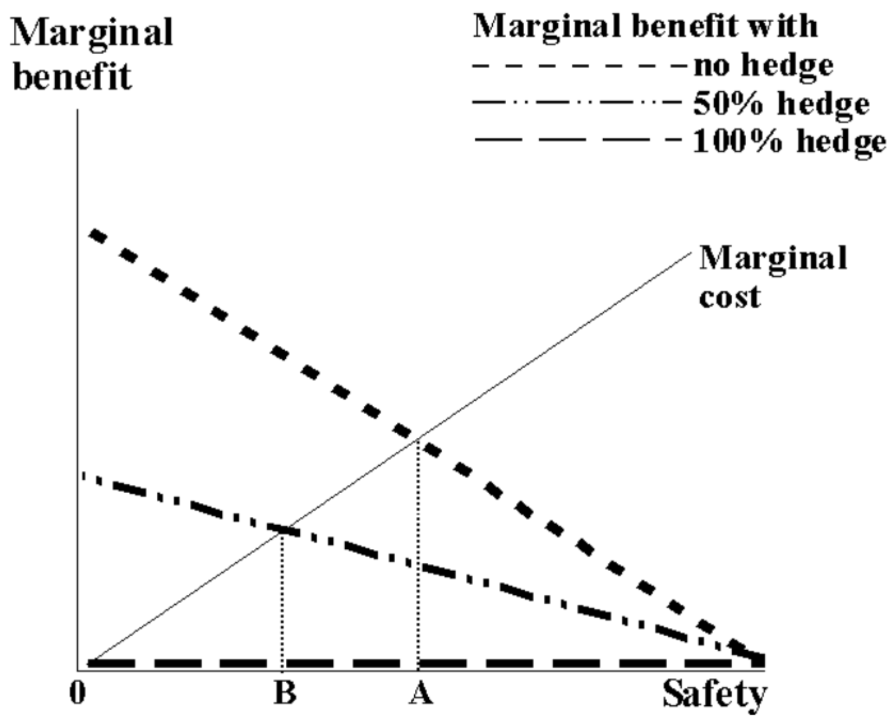

Figure 1. Level of safety as a function of the coinsurance rate for risk-insensitive insurance