Problem set 9 solution procedures and requirements

Problem Set 9 is essentially a reparameterized version of the problem that we worked on in class last Thursday (cf. pp. 6-8 in the Credit Risk lecture note).

In order to fully comprehend the pricing of credit risk in the Black-Scholes-Merton framework, it is important that students work the problem by hand, and only rely upon a spreadsheet model (either your own or the one that I have posted on the course website) in order to validate your work. The computation strategy for completing this problem set is best described as follows:

1. Calculate d_1 and d_2, where $d_1 = \dfrac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$. Since d_1 and d_2 represent critical values for the standard normal distribution, obtain $N(d_1)$ and $N(d_2)$ accordingly. Since $N(d_2)$ corresponds to the risk neutral probability that $F \geq B$ at date T, it follows that 1 - $N(d_2)$ corresponds to the risk neutral probability that $F < B$ at date T; i.e., this is the risk neutral probability that the firm defaults on its promised debt payment. Also, because of the symmetry of the standard normal distribution, 1 - $N(d_2) = N(-d_2)$.

2. Note that the value of risky debt, $V(D)$ corresponds to the value of safe debt (Be^{-rT}) minus the value of the limited liability put option $V(Max[0, B - F])$, where F is the terminal value of risky assets, B is the terminal (date T) value of a riskless zero coupon (AKA "pure discount") bond and $V(Max[0, B-F]) = Be^{-rT}(N(-d_2)) - V(F)(N(-d_1))$. Thus, the "fair market value for the bond" is determined by calculating $V(D) = Be^{-rT} - [Be^{-rT}(N(-d_2)) - V(F)(N(-d_1))]$. The dollar value of the limited liability put option is given by $V(Max[0, B - F]) = Be^{-rT}(N(-d_2)) - V(F)(N(-d_1))$, which also corresponds to the "fair premium" for credit insurance (cf. part 3 of Problem Set 9).

3. Problem Set 9 also asks for the yield to maturity and credit risk premium. The yield to maturity (YTM) for a T-period pure discount bond corresponds to the rate of interest which must be earned from date 0 to date T in order for the future value of $V(D)$ to be equal to B; i.e., $B = V(D)e^{YTM(T)}$. Solving for YTM in this equation, we find that $YTM = \ln(B/V(D))/T$. The credit risk premium corresponds to the difference between the yield to maturity (YTM) and the riskless rate of interest r. This risk premium compensates investors for bearing default risk costs. Intuitively, it makes a lot of sense that there is a positive relationship between the risk of default and the credit risk premium.