Solutions for the Risk Pooling Class Problem

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1. Consider an insurer which has sold 5 *independent* and *identically* distributed insurance policies, each having an expected loss of \$1,000 and a standard deviation of \$1,000. Furthermore, also assume that losses are *normally* distributed. Calculate i) the expected value and standard deviation of the insurer's average loss distribution, and ii) the probability that the loss on an average policy will exceed \$1,500.

   SOLUTION:

   1. $E(L_p) = \sum_{i=1}^{n} w_i E(L_i) = (1/n)n\mu = \mu = \underline{\textbf{\$1,000}}$, and $\sigma^2_{L_p} = \dfrac{\sigma^2}{n} + \dfrac{(n-1)}{n}\rho\sigma^2 = 1{,}000^2/5$ $+ 0 = 200{,}000$; $\therefore \sigma_{Lp} = \underline{\textbf{\$447.21}}$.
   2. Since losses are normally distributed and \$1,500 is 1.118 standard deviations greater than $\mu =$ \$1,000, there is a 13.18% probability that the loss on an average policy will exceed \$1,500.

2. Suppose the insurer described above decides to sell 10 such policies rather than just 5. Perform the same calculations as in the previous bullet point and explain the difference in your results.

   SOLUTION:

   1. $E(L_p) = \sum_{i=1}^{n} w_i E(L_i) = (1/n)n\mu = \mu = \underline{\textbf{\$1,000}}$, and $\sigma^2_{L_p} = \dfrac{\sigma^2}{n} + \dfrac{(n-1)}{n}\rho\sigma^2 = 1{,}000^2/10$ $+ 0 = 1{,}000{,}000$; $\therefore \sigma_{Lp} = \underline{\textbf{\$316.23}}$.
   2. Since losses are normally distributed and \$1,500 is 1.5811 standard deviations greater than $\mu =$ \$1,000, there is a 5.69% probability that the loss on an average policy will exceed \$1,500.

   DISCUSSION: Since losses are identically distributed, increasing the size of the risk pool has no effect upon the expected value of the insurer's average loss distribution. However, the standard deviation is reduced due to diversification. Since the standard deviation of the average policy is smaller than before, the probability that losses on average exceed a large amount is also reduced.

3. Suppose the insurer described above decides to sell 10 policies but now believes that the correlation between policies is .1 rather than zero. Compare your results with those obtained in the previous bullet point and explain any differences.

SOLUTION:

1. $E(L_p) = \sum_{i=1}^{n} w_i E(L_i) = (1/n)n\mu = \mu = \underline{\mathbf{\$1{,}000}}$, and $\sigma^2_{L_p} = \dfrac{\sigma^2}{n} + \dfrac{(n-1)}{n}\rho\sigma^2 = 1{,}000^2/10$ $+ (9/10)(.1)1{,}000^2 = 190{,}000$; $\therefore \sigma_{Lp} = \underline{\mathbf{\$435.89}}$.

2. Since losses are normally distributed and \$1,500 is 1.1471 standard deviations greater than $\mu = \$1{,}000$, there is a 12.57% probability that the loss on an average policy will exceed \$1,500.

DISCUSSION: Holding n constant, increasing ρ has no effect upon the expected value of the insurer's average loss distribution. However, for a given n, positively correlated risks are on average riskier than uncorrelated risks. In this case, increasing ρ from 0 to .1 causes the standard deviation of the average loss distribution to increase from \$316.23 to \$435.89. Consequently, the risk of extreme outcomes is greater compared with the previous case (where risks are statistically independent).