Consider the following question (sourced from page 6 of the Decision Making under Risk and Uncertainty, part 1 lecture note); Investments A and B are initially riskless, and the rate of interest is zero).

Q: Suppose you are considering two mutually exclusive, riskless investment choices that cost \$50 each; Investment A pays back \$100, whereas Investment B pays back \$120. Your initial wealth is \$50. Which investment should you choose?

A: If you were to take either investment, your funding comes from your initial wealth of \$50. Since the payoff on A is \$100, by investing in A your wealth increases by \$50 (which corresponds to the net present value (NPV) of investment A). On the other hand, since the payoff on B is \$120, by investing in B, your wealth increases by \$70. The logical choice (assuming that more wealth is preferred to less wealth) is to choose Investment B rather than Investment A.

On page 7 of the aforementioned lecture note, risk is introduced by replacing A's and B's riskless \$100 and \$120 payoffs with risky \$100 and \$120 *expected* payoffs. Specifically, Investment A pays off \$50 with probability .5 and \$150 with probability .5, whereas Investment B pays off \$0 with probability .5 and \$240 with probability .5. Although the expected value of the payoff on Investment B is greater than the expected value of the payoff on Investment A (\$120 versus \$100), Investment B is riskier than Investment A, since its standard deviation is \$120 compared with Investment A's standard deviation of \$50.

The decision whether to invest in A or B depends solely on risk preferences. If you happen to be a *risk loving* investor, then Investment B's higher standard deviation is a *positive* attribute; therefore, you prefer B over A. If you are a *risk neutral* investor, you prefer B over A because B has a higher expected value and you don't care about risk. However, if you are a *risk averse* investor, while you are attracted by B's higher expected value, you dislike its higher risk. Thus, if you are very risk averse, then you may prefer to give up expected value in order to bear less risk by investing in A. On the other hand, if you are somewhat less risk averse, then you might like to take on B's extra risk so that you can earn its higher expected payoff. Therefore, an assessment of the risk-reward tradeoff between these investments is called for, which is accomplished by calculating expected utility associated with investing in Investment A and compare A's expected utility with the expected utility associated with investing in Investment B.

Suppose $U(W) = W^{.5}$. Then

$$\begin{aligned} E(U(W_A)) &= .5(50^{.5}) + .5(150^{.5}) = .5(7.071) + .5(12.247) = 9.659, \text{and} \\ E(U(W_B)) &= .5(0^{.5}) + .5(240^{.5}) = .5(0) + .5(15.492) = 7.746. \end{aligned}$$

Since expected utility is higher for A than for B, then the less risky investment (Investment A) is chosen. Apparently, an investor with square root utility is sufficiently risk averse that she prefers to forgo B's higher expected value in order to bear A's lower level of risk. However, suppose another investor has $U(W) = W^{.8}$. Then

$$\begin{aligned} E(U(W_A)) &= .5(50^{.8}) + .5(150^{.9}) = .5(22.865) + .5(55.065) = 38.965, \text{and} \\ E(U(W_B)) &= .5(0^{.8}) + .5(240^{.8}) = .5(0) + .5(80.199) = 40.010. \end{aligned}$$

This latter investor is less risk averse than the former investor, and prefers Investment B over A.