BAYLOR UNIVERSITY
HANKAMER SCHOOL OF BUSINESS
DEPARTMENT OF FINANCE, INSURANCE & REAL ESTATE

Risk Management
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Midterm Exam #2
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Name ___SOLUTIONS_________

Problem 1

An entrepreneur has initial wealth of \$880. Her initial wealth is invested in two factories, each of which is worth \$400. Her remaining \$80 in initial wealth is invested in cash. Each factory has a 25% chance of being destroyed and a 75% chance of not suffering any damage. Because the factories are located far away from each other, these risks are statistically independent.

Since the entrepreneur has \$80 in cash, she can use some or all of this money to purchase actuarially fair insurance policies to cover her risks. Note that the price for an actuarially fair insurance policy equals the expected value of the payoff (indemnity) provided by the insurance policy.

A. (8 points) Given the entrepreneur's cash resources, if she covers 60% of the first factory's potential loss, what is the maximum level of coverage (in terms of proportion of potential loss) that she can purchase against the risk that the second factory will be destroyed?

   SOLUTION: The expected value of each factory's potential loss is $E(L) = \sum_{s=1}^{n} p_s L_s = .25(400)$ = \$100. Since the price for an actuarially fair insurance policy equals the expected value of the payoff (indemnity) provided by the insurance policy, this means that the premium for the first factory's insurance policy is $.6\sum_{s=1}^{n} p_s L_s$ = (.6).25(400) = \$60. Since the entrepreneur has \$80 in cash, the maximum she can pay to insure the second factory is \$20, which implies a maximum level of coverage of 20%.

B. (8 points) Given the entrepreneur's cash resources, what is the maximum level of coverage (in terms of proportion of potential loss) for each factory that will result in the same premium being paid for each policy?

   SOLUTION: Since the most that the entrepreneur can spend on insurance is \$80 and the expected value of each factory's potential loss is \$100, this means that she can cover 40% of the potential loss for each factory for a premium of \$40 per factory.

C. (8 points) Suppose the entrepreneur's utility function is $U(W) = \ln W$. Show that the entrepreneur is better off if she insures both factories at the same level of coverage (for a total premium of \$80) than she would be if she implemented the risk management strategy implied in Part A of this problem.

SOLUTION: Since the risks are statistically independent, this implies that the joint probability distribution for both risks consists of 4 possible states of the world: 1) no loss on either factory (with probability .75 x .75 = 56.25%), 2) losses on both factories (with probability .25 x .25 = 6.25%), 3) loss on factory 1 and no loss on factory 2 (with probability .75 x .25 = 18.75%) and 4) no loss on factory 1 and loss on factory 2 (with probability .25 x .75 = 18.75%). Furthermore, we need to derive an equation for state-contingent wealth in each of these states. Under this scenario (where she covers 40% of the potential loss for each factory for a premium of \$40 per factory), this implies that state-contingent wealth is $W_s = W_0 - \alpha_1 p_1 - \alpha_2 p_2 - (1-\alpha_1)L_{1s} - (1-\alpha_2)L_{2s} = 88 - 8 - .6L_{1s} - .6L_{2s}$. Thus the following distribution of state-contingent wealth is implied by this equation:

State	***p_s***	***L_{1s}***	***L_{2s}***	***W_s***	***$U(W_s)$***
no loss on either factory	56.25%	0	0	800	6.6846
losses on both factories	6.25%	400	400	320	5.7683
loss on factory 1 and no loss on factory 2	18.75%	400	0	560	6.3279
no loss on factory 1 and loss on factory 2	18.75%	0	400	560	6.3279
	Expected Value:	**100**	**100**	**680**	**6.4936**

Thus the expected utility of this risk management decision is 6.4936. Now suppose that the entrepreneur implements the risk management decision implied in part A. In other words, she covers 60% of the first factory's potential loss for a premium of \$6 and 20% of the second factory's potential loss for a premium of \$2. This risk management decision results in the following distribution of state-contingent wealth:

State	***p_s***	***L_{1s}***	***L_{2s}***	***W_s***	***$U(W_s)$***
no loss on either factory	56.25%	0	0	800	6.6846
losses on both factories	6.25%	400	400	320	5.7683
loss on factory 1 and no loss on factory 2	18.75%	400	0	640	6.4615
no loss on factory 1 and loss on factory 2	18.75%	0	400	480	6.1738
	Expected Value:	**100**	**100**	**680**	**6.4897**

Thus the expected utility of this alternative risk management decision is 6.4897, which implies that she is better off if she insures both factories at the same level of coverage.

D. (8 points) Explain *why* the expected utility of having the same level of coverage on both factories is higher than the expected utility of having different levels of coverage.

SOLUTION: This raises an interesting question - why is the expected utility of having the same level of coverage on both factories higher than the expected utility of having different levels of coverage? The answer is quite simple. We know from the expected utility theory that a mean preserving spread will always produce a lower expected utility ranking. In this problem, the alternative risk management decision involving different levels of coverage represents a mean preserving spread of the risk management decision involving the same level of coverage on both factories. Looking closer, the source of the greater dispersion associated with the alternative risk management decision occurs whenever one factory is destroyed and the other factory doesn't suffer any damage. Comparing these tables, state-contingent wealth in the 3rd and 4th states varies when there are different levels of coverage, but is the same when the same level of coverage is selected for both factories.

Problem 2

Suppose that you are considering investing in a portfolio consisting of two securities, A and B. You have estimated that these two securities will provide the following set of state-contingent returns ($r_{A,s}$ and $r_{B,s}$), depending on how well or poorly the economy performs (note: p_s represents the probability that state s will occur):

State of Economy	p_s	$r_{A,s}$	$r_{B,s}$
Boom	50%	30%	3%
Bust	50%	0%	9%

A. (8 points) What are the expected returns for securities A and B?

SOLUTION:

$$E(r_A) = \sum_{s=1}^{n} p_s r_{As} = .5(.30) + .5(0) = 15\%.$$

$$E(r_B) = \sum_{s=1}^{n} p_s r_{Bs} = .5(.03) + .5(.09) = 6\%$$

B. (8 points) What are the standard deviations of the returns for securities A and B?

SOLUTION:

Since there are equal probabilities of Boom and Bust, the standard deviation of each security can also be determined by simply calculating ½ of the total dispersion between the Boom and Bust states; thus, $\sigma_A = ½(.30 - .0) = 15\%$, and $\sigma_B = ½(.09 - .06) = 3\%$.

C. (16 points) Suppose that state contingent returns on the risk free asset ($r_{f,s}$) and the market portfolio ($r_{m,s}$) are estimated as follows:

State of Economy	p_s	$r_{f,s}$	$r_{m,s}$
Boom	50%	6%	20%
Bust	50%	6%	5%

Using this information in conjunction with the information in Problem 3 on Securities A and B, are Securities A and B underpriced, overpriced, or correctly priced? Justify your answer.

SOLUTION: In order to determine whether Securities A and B are underpriced, overpriced, or correctly priced, we need to calculate the beta coefficients for each security. Since $\beta_i = \sigma_{im} / \sigma_m^2$, this requires calculating the covariances between A and the market and B and the market, and then dividing each of these numbers through by the variance of the market.

First, we find the variance of the market. Since there are equal probabilities of Boom and Bust, the standard deviation of the market is $\sigma_m = ½(.20 - .05) = 7.5\%$; therefore $\sigma_m^2 = 0.563\%$.

Next, we find the covariances; since $E(r_m)$ = .5(.20) + .5(.05) = .125,

$$\sigma_{Am} = \sum_{s=1}^{n} p_s(r_{As} - E(r_A))(r_{ms} - E(r_m)) = .5(.30-.15)(.20-.125) + .5(0-.15)(.05-.125) = 0.0113$$
$$\sigma_{Bm} = \sum_{s=1}^{n} p_s(r_{Bs} - E(r_B))(r_{ms} - E(r_m)) = .5(.03-.06)(.20-.125) + .5(.09-.06)(.05-.125) = -.0023$$

Therefore,

$$\beta_A = \frac{\sigma_{Am}}{\sigma_m^2} = \frac{.0113}{.00563} = 2, \text{ and } \beta_B = \frac{\sigma_{Bm}}{\sigma_m^2} = \frac{-.0023}{.00563} = -.4.$$

Consequently,

$$E(r_A) = r_f + \left[E(r_m) - r_f\right]\beta_A = 6\% + [12.5\% - 6\%](2) = 19\%, \text{ and}$$
$$E(r_B) = r_f + \left[E(r_m) - r_f\right]\beta_B = 6\% + [12.5\% - 6\%](-.4) = 3.4\%.$$

Since security A should provide an expected return of 19% but is currently priced to provide an expected return of 15%, this means that this security is overpriced. Investors will realize this, and consequently there will be excess supply for security A, which will cause its price to decrease and its expected return to rise until it is in line with the CAPM. Conversely, security B should provide an expected return of only 3.4% but is currently priced to provide an expected return of 6%, which means that this security is underpriced. Investors will realize this, and consequently there will be excess demand for security B, which will cause its price to increase and its expected return to fall until it is in line with the CAPM.