

Portfolio Theory Class Problem

Finance 4335, March 24, 2020

Consider a market with only two securities, numbered 1 and 2. Expected returns are $E(r_1) = 12\%$ and $E(r_2) = 8\%$, standard deviations are $\sigma_1 = 10\%$ and $\sigma_2 = 4\%$, and $\rho_{12} = -1$; i.e., returns are perfectly negatively correlated.

What is the expected return and standard deviation for the minimum risk combination of securities 1 and 2?

SOLUTION: The weights for the minimum risk combination of 2 securities can be determined

using the following equations: $w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$ and $w_2 = 1 - w_1$. Therefore, $w_1 =$

$$\frac{16 + 40}{100 + 16 + 80} = \frac{56}{196} = \frac{2}{7}, \text{ and } w_2 = 1 - w_1 = \frac{5}{7}. \text{ Consequently,}$$

$$E(r_p) = w_1(.12) + w_2(.08) = (2/7)(.12) + (5/7)(.08) = 9\frac{1}{7}\%, \text{ and}$$

$$\sigma_p = \sqrt{w_1^2(.1^2) + w_2^2(.04^2) - w_1w_2(.08)} = \sqrt{\frac{4}{49}(.01) + \frac{25}{49}(.016) - \frac{10}{49}(.08)} = 0.$$