Pricing Credit Risk and Problem Set 9 Solution Procedure

This brief teaching note explains the solution procedure for the "Pricing Credit Risk" numerical example (see pp. 6-9 of the "Credit Risk" lecture note), and for Problem Set 9.

In order to fully comprehend the pricing of credit risk in the Black-Scholes-Merton framework, I recommend that students work each problem by hand, and also create a spreadsheet model in order to validate their work. The computation strategy for completing both of these problems is as follows:

1. **Where to Begin**: Start by calculating d_1 and d_2, where $d_1 = \dfrac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$. Since d_1 and d_2 represent critical values for the standard normal distribution, obtain $N(d_1)$ and $N(d_2)$ accordingly. Since $N(d_2)$ corresponds to the risk neutral probability that $F \geq B$ at date T, it follows that 1 - $N(d_2)$ corresponds to the risk neutral probability that $F < B$ at date T; i.e., this is the risk neutral probability that the firm defaults on its promised debt payment. Also, because of the symmetry of the standard normal distribution, expressions such as $1 - N(d_1)$ and $1 - N(d_2)$ can be equivalently written as $N(-d_1)$ and $N(-d_2)$ respectively.

2. **Fair Market Value for a Risky Bond**: Note that the value of risky debt, $V(D)$, corresponds to the value of safe debt ($V(B) = Be^{-rT}$) minus the value of the limited liability put option ($V(Max[0, B - F])$), where F is the terminal value of risky assets, B is the terminal (date T) value of a riskless zero coupon (also known as a "pure discount") bond and $V(Max[0, B - F]) = Be^{-rT}(N(-d_2)) - V(F)(N(-d_1))$. Thus, the "fair market value for the bond" is determined by calculating $V(D) = Be^{-rT} - [Be^{-rT}(N(-d_2)) - V(F)(N(-d_1))]$. The dollar value of the limited liability put option is given by $V(Max[0, B - F]) = Be^{-rT}(N(-d_2)) - V(F)(N(-d_1))$, which also corresponds to the "fair premium" for credit insurance.

3. **Yield to Maturity and Credit Risk Premium**: The yield to maturity (YTM) for a T-period pure discount bond corresponds to the rate of interest which must be earned from date 0 to date T in order for the future value of $V(D)$ to be equal to B; i.e., $B = V(D)e^{YTM(T)}$. Solving for YTM in this equation, we find that $YTM = \ln(B/V(D))/T$. The credit risk premium corresponds to the difference between the yield to maturity (YTM) and the riskless rate of interest r. This risk premium compensates investors for bearing default risk costs. Intuitively, it makes a lot of sense that there is a positive relationship between the risk of default and the credit risk premium.