**Risk Aversion Class Problem Solutions**

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Individual #1 has the following utility function: \( U(W) = \sqrt{W} \). Her initial wealth is $10 and she is offered a coin toss which pays off $6 if the coin comes up heads and -$6 if the coin comes up tails.

A. Compute the exact value of the certainty equivalent and of the risk premium for #1.

**SOLUTION:** Solving for the “exact” values of the certainty equivalent \( W_{CE} \) and the risk premium \( \lambda(E(W)) \) requires that we first find the expected utility of the gamble, set this equal to the utility of the certainty equivalent, and then compute \( W_{CE} \) directly. Once we know \( W_{CE} \), then \( \lambda(E(W)) = E(W) - W_{CE} \). Since the state space is \((4,16),(5,5)\), \( E(W) = 10 \) and \( E(U(W)) = .5(2)+.5(4) = 3 \). Therefore, \( E(U(W)) = \sqrt{W_{CE}} = 3 \); thus \( W_{CE} = 9 \), and \( \lambda(E(W)) = 1 \).

B. Apply the Arrow-Pratt absolute risk aversion formula to obtain an approximation of the risk premium for #1.

**SOLUTION:** The “approximate” value of \( \lambda(E(W)) \approx \sigma^2 R_A(E(W)) \), where \( R_A(E(W)) \) corresponds to the ratio \(-U''(W)/U'(W)\) evaluated at \( E(W) \). For this utility function, \( U'(W) = .5W^{-1.5} \) and \( U''(W) = -.25W^{-1.5} \), so \(-U''(W)/U'(W) = .5/W \) and \( R_A(E(W)) = .5/10 = .05 \). Since the standard deviation of a fair coin toss is half of the total dispersion between the state contingent wealth values, this implies that \( \sigma = 6 \), which implies that \( \sigma^2 = 36 \). Therefore, \( \lambda(E(W)) \approx 36(.5)(.05) = .9 \).

C. Show that #1’s absolute risk aversion is decreasing in wealth.

**SOLUTION:** Since \( R_A(W) = .5/W \), \( \frac{dR_A(W)}{dW} = R_A(W) = -.5/W^2 \); i.e., #1’s absolute risk aversion is decreasing in wealth. Although the calculus lends a nice touch, that #1’s absolute risk aversion is decreasing in wealth is apparent by inspection.

D. Suppose that individual #2 is offered this gamble. Individual #2 is identical in all respects to individual #1, except #2’s utility \( U(W) = W^{.25} \). Compute the exact value of the certainty equivalent and of the risk premium for #2, and also apply the Arrow-Pratt absolute risk aversion formula to obtain an approximation of the risk premium for this individual.

**SOLUTION:** Individual #2’s expected utility \( E(U(W)) = .5(4^{.25})+.5(16^{.25}) = 1.707 \). Therefore, \( E(U(W)) = W^{.25}_{CE} = 1.707 \); thus \( W_{CE} = 1.707^4 = 8.49 \), and the “exact” \( \lambda(E(W)) = 1.51 \). Since \( R_A(W) = .75/W \) for Individual #2, it follows that \( \lambda(E(W)) \approx 36(.5)(.075) = 1.35 \).

E. Who is more risk averse, #1 or #2? Explain why.

**SOLUTION:** Individual #2 is more risk averse than Individual #1, since #2 has a higher risk aversion coefficient than #1. Consequently, other things equal, #2 also has a higher risk premium.