Insurance Economics Class Problem Helpful Hints

Finance 4335, February 25, 2023

Suppose that a consumer is subject to the following loss distribution:

State-Contingent Loss (L_s)	Probability of State (p_s)
\$0	1/3
\$2,500	1/3
\$5,000	1/3

This consumer is considering four possible strategies for dealing with this risk. Besides self-insurance, she can also consider the following three insurance policies:

a) Policy A has a \$625 deductible for a premium of \$2,375;
b) Policy B covers 80% of all losses for a premium of \$2,250; and
c) Policy C covers 100% of all losses for a premium of \$3,000.

A. Suppose the consumer's initial wealth $W_0 =$ \$10,000, and the only source of risk is the loss distribution. Calculate the expected value of wealth under the four available risk management strategies (i.e., self-insurance, Policy A, Policy B, and Policy C).

   HINTS for part A: In order to determine the expected value of wealth under the four available risk management strategies, we need to know what state-contingent wealth is in each of the three states for each of the four strategies.

   - Under self-insurance, since the consumer retains all risk, it follows that state-contingent wealth is equal to initial wealth minus state-contingent loss in each state; i.e., $W_s = W_0 - L_s$.
   - Under Policy A, the consumer pays up to the first \$625 of loss, and the insurer covers any loss in excess of \$625. Therefore, state-contingent wealth under Policy A ($W_{A,s}$) equals initial wealth W_0 minus the insurance premium P_A paid for Policy A, less the deductible d which is paid only if there are losses; i.e., $W_{A,s} = W_0 - P_A - (L_s - \max(0, L_s - d))$, or equivalently, $W_{A,s} = W_0 - P_A - \min(L_s, d)$.
   - Under Policy B, the consumer's state-contingent wealth is equal to initial wealth W_0 minus the insurance premium P_B paid for Policy B, minus the uninsured loss. Since Policy B covers $\alpha = 80\%$ of losses, it follows that the uninsured loss $1 - \alpha = 20\%$; i.e., $W_{B,s} = W_0 - P_B - (1 - \alpha)L_s$.
   - Under Policy C, the consumer's state-contingent wealth is equal to initial wealth W_0 minus the insurance premium P_C paid for Policy C. Since $\alpha = 100\%$ under full coverage, it follows that $W_{C,s} = W_0 - P_C$ for all s.

B. What are the premium loadings for Policies A, B, and C?

   HINT for part B:

- The premium loading corresponds to the difference between the premium charged by the insurer for each policy and its actuarially fair value. Actuarially fair value is, by definition, the *expected value of the indemnity* offered by the insurer under a given policy.
- As I explain in my Demand for Insurance: Full vs. Partial Coverage" assigned reading, an insurance policy is a contract of indemnity under which the insurer agrees to pay claims according to an indemnity schedule. As shown on page 1 of that reading, the state-contingent indemnity schedule I_s for Policy A is given by $I_s = \max(0, L_s - d)$, where $d = \$625$. For Policy B, the state-contingent indemnity schedule $I_s = \alpha L_s$, where $\alpha = .80$. For Policy C, since $\alpha = 100\%$, the state-contingent indemnity schedule $I_s = L_s$.
- Therefore, in order to determine the premium loadings for Policies A-C, one begins by calculating $E(I_A), E(I_B)$, and $E(I_C)$. Then the premium loadings (in terms of dollar markups) for Policies A-C are $P_A - E(I_A), P_B - E(I_B)$ and $P_C - E(I_C)$. Since premium loadings are often expressed in terms of percentage markups; the loadings (in terms of percentage markups) for Policies A-C are $[P_A - E(I_A)] / E(I_A), [P_B - E(I_B)] / E(I_B)$, and $[P_C - E(I_C)] / E(I_C)$.

C. Suppose that $U(W) = \ln W$. Which risk management strategy (i.e., self-insurance, Policy A, Policy B, or Policy C) should be selected?

HINT for part C: Since we know from part A what the state-contingent wealth is under each risk management strategy, all that remains to be done is to calculate expected utilities for all 4 strategies and select the strategy which yields the highest expected utility.